Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

4005973656

FURTHER MATHEMATICS **9231/23**

Paper 2 Further Pure Mathematics 2 **May/June 2020**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

DC (LK/CB) 187469/1

1 Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} - 8\frac{dx}{dt} - 9x = 9e^{8t}.$$ [6]

2 Let $I_n = \int_0^1 (1+3x)^n e^{-3x}\,dx$, where n is an integer.

(a) Show that $3I_n = 1 - 4^n e^{-3} + 3nI_{n-1}$. [3]

....

....

....

....

....

....

....

....

....

....

....

....

(b) Find the exact value of I_2. [3]

....

....

....

....

....

....

....

....

....

....

....

....

....

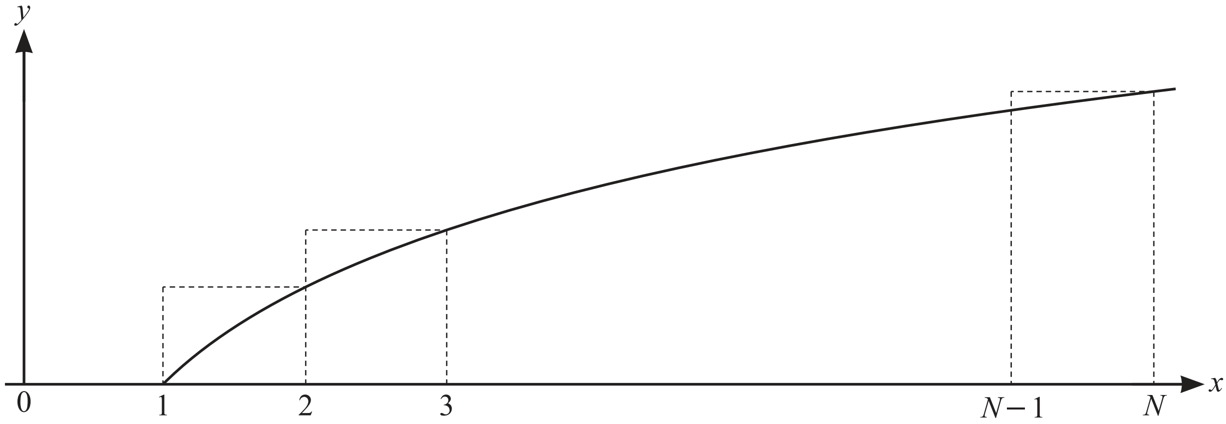
3 The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 5 & -1 & 7 \\ 0 & 6 & 0 \\ 7 & 7 & 5 \end{pmatrix}.$$

(a) Find the eigenvalues of **A**. [4]

(b) Use the characteristic equation of **A** to find $\mathbf{A}^{-1}$. [4]

4



The diagram shows the curve with equation $y = \ln x$ for $x \geqslant 1$, together with a set of $(N-1)$ rectangles of unit width.

(a) By considering the sum of the areas of these rectangles, show that

$$\ln N! > N \ln N - N + 1.$$ [5]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(b) Use a similar method to find, in terms of N, an upper bound for $\ln N!$. [3]

5 The curve C has parametric equations

$$x = \frac{1}{2}t^2 - \ln t, \qquad y = 2t + 1, \qquad \text{for } \frac{1}{2} \leqslant t \leqslant 2.$$

(a) Find the exact length of C. [5]

(b) Find $\frac{d^2y}{dx^2}$ in terms of t, simplifying your answer. [4]

6 **(a)** Starting from the definitions of tanh and sech in terms of exponentials, prove that

$$1 - \tanh^2\theta = \operatorname{sech}^2\theta.$$ [3]

....

....

....

....

....

....

....

....

The variables x and y are such that $\tanh y = \cos\left(x + \frac{1}{4}\pi\right)$, for $-\frac{1}{4}\pi < x < \frac{3}{4}\pi$.

(b) By differentiating the equation $\tanh y = \cos\left(x + \frac{1}{4}\pi\right)$ with respect to x, show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\operatorname{cosec}\left(x + \frac{1}{4}\pi\right).$$ [4]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(c) Hence find the first three terms in the Maclaurin's series for $\tanh^{-1}\left(\cos(x+\frac{1}{4}\pi)\right)$ in the form $\frac{1}{2}\ln a + bx + cx^2$, giving the exact values of the constants a, b and c. [5]

7 **(a)** Show that an appropriate integrating factor for

$$(x^2+1)\frac{\mathrm{d}y}{\mathrm{d}x}+y\sqrt{x^2+1}=x^2-x\sqrt{x^2+1}$$

is $x+\sqrt{x^2+1}$. [4]

(b) Hence find the solution of the differential equation

$$(x^2+1)\frac{dy}{dx}+y\sqrt{x^2+1}=x^2-x\sqrt{x^2+1}$$

for which $y=\ln 2$ when $x=0$. Give your answer in the form $y=f(x)$. [7]

8 **(a)** Use de Moivre's theorem to show that $\sin^6\theta = -\frac{1}{32}(\cos 6\theta - 6\cos 4\theta + 15\cos 2\theta - 10)$. [6]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

It is given that $\cos^6\theta = \frac{1}{32}(\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10)$.

(b) Find the exact value of $\int_0^{\frac{1}{3}\pi} \left(\cos^6(\tfrac{1}{4}x) + \sin^6(\tfrac{1}{4}x)\right) \mathrm{d}x$. [4]

....

....

....

....

....

....

....

....

....

(c) Express each root of the equation $16c^6 + 16(1-c^2)^3 - 13 = 0$ in the form $\cos k\pi$, where k is a rational number. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....